The Breaking of Isospin and the ρ–ω–System

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Simple quark models for the low lying vector mesons suggest a mixing between the u– and d–flavors and a violation of the isospin symmetry for the $\rho - \omega$ system much stronger than observed. It is shown that the chiral dynamics, especially the QCD anomaly, is responsible for a restoration of the isospin symmetry in the $\rho - \omega$ system.

Although there are no doubts that all observed strong interaction phenomena can be described within the theory of QCD, a quantitative description of the strong interaction phenomena in the low energy sector is still lacking, although some features of the low energy phenomena have been partially understood by the lattice gauge theory approach.

The low energy sector of the physics of the strong interactions is dominated by the low–lying pseudoscalar mesons (π, K, η, η') and the low–lying vector mesons $(\rho, \omega, K^*, \phi)$. It is well–known that the structures of the quark wave functions of the pseudoscalar mesons (0^{-+}) and of the vector mesons (1^{--}) differ substantially.

In the vector meson channel there is a strong mixing between the eights component of the $SU(3)$ octet (wave function: $(\bar{u}u + \bar{d}d - 2\bar{s}s)/(\sqrt{6})$) and of the $SU(3)$ singlet (wave function: $(\bar{u}u + \bar{d}d + \bar{s}s)/(\sqrt{3})$). The mixing strength is such that the mass eigenstates are nearly the state $(\bar{u}u + \bar{d}d)/(\sqrt{2})$), the ω–meson, and the state $\bar{s}s$, the ϕ–meson. While this feature looks peculiar, when viewed upon from the platform of the underlying $SU(3)$ symmetry, it finds a simple interpretation, if one takes into account the Zweig rule [1], which states that the mixing must take place in such a way that quark lines are neither destroyed nor created.

On the other hand the pseudoscalar mesons follow the pattern prescribed by the $SU(3)$ symmetry in the absence of singlet–octet mixing. The neutral

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(1)

mass eigenstates η and η' are nearly an $SU(3)$–octet or $SU(3)$–singlet:

$$\eta \approx \frac{1}{\sqrt{6}} \left(\bar{u}u + \bar{d}d - 2\bar{s}s \right) \quad \text{or} \tag{1}$$

$$\eta' \approx \frac{1}{\sqrt{3}} \left(\bar{u}u + \bar{d}d + \bar{s}s \right).$$

This indicates a large violation of the Zweig rule in the 0^{-+} channel [2] [3]. Large transitions between the various $(\bar{q}q)$–configurations must take place. In QCD the strong mixing effects are related to the spontaneous breaking of the chiral $U(1)$ symmetry normally attributed to instantons. Effectively the mass term for the pseudoscalar mesons can written as follows, neglecting the effects of symmetry breaking in the gluonic mixing term [4] [5] [6]:

$$M_{\bar{q}q}^2 = \begin{pmatrix} M_u^2 & 0 & 0 \\ 0 & M_d^2 & 0 \\ 0 & 0 & M_s^2 \end{pmatrix} + \lambda \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{2}$$

where M_u^2, M_d^2 and M_s^2 are the M^2–values of the masses of quark composition $\bar{u}u, \bar{d}d$ and $\bar{s}s$ respectively.

It is well–known that the mass and mixing pattern of the 0^{-+}–mesons is described by such an ansatz [2]. The parameter λ, which describes the mixing strength due to the gluonic forces, is essentially given by the η'–mass: $\lambda \cong 0.24$ GeV^2. Since λ is large compared to the strength of $SU(3)$ violation given by the s–quark mass, large mixing phenomena are present in the 0^{-+} channel, as seen in the corresponding wave functions.

The situation is different in the vector meson 1^{--} channel. Here the gluonic mixing term is substantially smaller than the strength of $SU(3)$ violation such that the Zweig rule is valid to a good approximation. If one describe the mass matrix for the vector mesons in a similar way as for the pseudoscalar, we have

$$M_{\bar{q}q} = \begin{pmatrix} M(\bar{u}u) & 0 & 0 \\ 0 & M(\bar{d}d) & 0 \\ 0 & 0 & M(\bar{s}s) \end{pmatrix} + \tilde{\lambda} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{3}$$

here $M(\bar{q}q)$ denotes the mass of a vector meson with quark composition $\bar{q}q$ in the absence of the mixing term. The magnitude of the mixing term $\tilde{\lambda}$ can be obtained in a number of different ways, e.g by considering the ρ_0–ω mass difference. Neglecting the isospin violation caused by the m_d–m_u mass splitting, the gluonic mixing term is responsible for the ρ_0–ω mass shift:

$$M_\omega - M_\rho = 2\tilde{\lambda} \quad , \tag{4}$$

$$\tilde{\lambda} \cong 6.0 \pm 0.5 \text{ MeV} .$$

In QCD the isospin symmetry is violated by the mass splitting between the u– and d–quark. Typical estimates give:

$$\frac{m_d - m_u}{\frac{1}{2}(m_d + m_u)} \cong 0.58 . \qquad (5)$$

The observed smallness of isospin breaking effects is usually attributed to the fact that the mass difference $m_d - m_u$ is small compared to the QCD scale Λ_{QCD}. However in the case of the vector mesons the QCD interaction enters in two different ways:

a)   In the chiral limit of vanishing quark masses the masses of the vector mesons are solely due to the QCD interaction, i.e. $M = \text{const} \cdot \Lambda_{QCD}$.

b) The QCD mixing term will lead to a mixing among the various flavour components such that the $SU(3)$ singlet (quark composition $(\bar{u}u + \bar{d}d + \bar{s}s)/\sqrt{3}$) is lifted upwards compared to the two other neutral components given by the wave functions $(\bar{u}u - \bar{d}d)/\sqrt{2}$ and $(\bar{u}u + \bar{d}d - 2\bar{s}s)/\sqrt{6}$. The corresponding mass shift is given by $3\tilde{\lambda}$.

We approach the real world by first introducing the mass of the strange quark. As soon as m_s becomes larger than $3\tilde{\lambda}$, substantial singlet–octet mixing sets in, and the mass of one vector meson increases until it reaches the observed value of the ϕ–mass. At the same time the Zweig rule, which is strongly violated in the chiral $SU(3)_L \times SU(3)_R$ limit becomes more and more valid.
The validity of the Zweig rule is determined by the ratio $m_s/\tilde{\lambda}$. If this ratio vanishes, the Zweig rule is violated strongly. In reality, taking m_s (1GeV) $\approx$ 150 MeV, the ratio $m_s/\tilde{\lambda}$ is about 25 implying that the Zweig rule is nearly exact.
In a second step we introduce the light quark masses m_u and m_d. We concentrate on the non–strange vector mesons. If the gluonic mixing interaction were turned off, the mass eigenstates would be $v_u = |\bar{u}u\rangle$ and $v_d = |\bar{d}d\rangle$. The masses of these mesons are given by:

$$M(v_u) = \langle v_u| H^0 + m_u \, \bar{u}u \, | v_u\rangle , \qquad (6)$$

$$M(v_d) = \langle v_d| H^0 + m_d \, \bar{d}d \, | v_d\rangle .$$

Here H^0 is the QCD–Hamiltonian in the chiral limit $m_u = m_d = 0$. Thus the masses can be written as

$$M(v_u) = M_0 + 2m_u \cdot c , \qquad (7)$$

$$M(v_d) = M_0 + 2m_d \cdot c \,.$$

(c: constant, given by the expectation value of $\bar{q}q$). The introduction of the light quark masses induces positive mass shifts for both v_u and v_d. These mass shifts can be estimated by considering the corresponding mass shifts of the charged K^*–mesons. One finds [7] [8]:

$$\begin{aligned} M(v_d) - M(v_u) &\cong 2\,(m_d - m_u) \cdot c \\ &\cong 1.7 \text{ MeV.} \end{aligned} \tag{8}$$

It is remarkable that this mass shift is of similar order of magnitude as the mass shift between the isosinglet and isotriplet state in the chiral limit, where isospin symmetry is valid. This implies that the strength of the gluonic mixing term is comparable to the $\Delta I = 1$ mass term. If follows that the eigenstates of the mass operator taking both the violation of isospin and the gluonic mixing into account will not be close to being eigenstates of the isospin symmetry.

For the ρ_0–ω system the mass operator takes the form:

$$M = \left(\begin{array}{cc} M(v_u) & 0 \\ 0 & M(v_d) \end{array} \right) + \tilde{\lambda} \left(\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right) \,. \tag{9}$$

Using $M(v_u) = M(\bar{u}u), M(v_d) = M(\bar{d}d)$ and $\tilde{\lambda} = 5.9$ MeV, we find

$$|\rho_0\rangle = 0.997 \Big| \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)\rangle + 0.071 \Big| \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\rangle$$

$$|\omega\rangle = -0.071 \Big| \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d)\rangle + 0.997 \Big| \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\rangle$$

The mixing angle α discribing the strength of the triplet–singlet mixing is about -4.1^o, i.e. a sizeable violation of isospin symmetry is obtained. Neither is the ρ_0–meson an isospin triplet, nor is the ω–meson an isospin singlet.

The conclusions we have derived follow directly from the observed smallness of the gluonic mixing in the vector meson channel and the $m_u - m_d$ mass splitting, as observed e.g. in the mass spectrum of the K^*–mesons. Nevertheless they are in direct conflict with observed facts. According to eq. (13), the probability of the ρ_0–meson to be an $I = |\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)\rangle$–state is $\sin^2 \alpha \cong 0.51\%$. Taking into account the observed branching ratio for the decay $\omega \to \pi^+\pi^-$, BR $\cong (2.21 \pm 0.30)\%$, this probability is bound to be less than 0.12%, in disagreement with the value derived above. Obviously our theoretical estimate cannot be correct.

We consider the discrepancy described above as a serious challenge for our understanding of the low energy sector of QCD.

We conclude: The mass difference $\Delta M = M(v_d) - M(v_u)$ must be smaller than estimated above. In order to reproduce the observed branching ratio for the decay $\omega \to \pi^+\pi^-$, ΔM cannot exceed 0.82 MeV, implying that our simple estimates based on quark–model considerations, cannot be correct. This can be seen as follows. We consider the following two–point functions

$$
\begin{aligned}
u_{\mu\nu} &= \langle 0|\bar{u}(x)\gamma_\mu u(x)\,\bar{u}(y)\gamma_\nu u(y)|0\rangle\,, \\
d_{\mu\nu} &= \langle 0|\bar{d}(x)\gamma_\mu d(x)\,\bar{d}(y)\gamma_\nu d(y)|0\rangle \\
m_{\mu\nu} &= \langle 0|\bar{d}(x)\gamma_\mu d(x)\,\bar{u}(y)\gamma_\nu u(y)|0\rangle\,.
\end{aligned}
\tag{10}
$$

The mixed spectral function $m_{\mu\nu}$ is expected to be essentially zero in the low energy region, since the two different currents can communicate only via intermediate gluonic mesons. In perturbative QCD these states would be represented by three gluons. The vanishing of $m_{\mu\nu}$ implies the validity of the Zweig rule.

The spectral functions $u_{\mu\nu}$ and $d_{\mu\nu}$ are strongly dominated at low energies by the ρ_0– and ω–resonances. The actual intermediate states contributing to the two-point functions are 2π–and 3π–states. However, a violation of the isospin symmetry due to the $u-d$–quark mass splitting does not show up in the π-meson spectrum. The $\pi^+ - \pi^\circ$ mass splitting is solely due to the electromagnetic interaction. It follows that resonant (2π) of (3π) states, i. e. the ρ–ω–resonances, cannot display the effects of the isospin violation either, and the mass difference $\Delta M = M(v_d) - M(v_u)$ must be very small.

Although the isospin symmetry is broken explicitly by the $u-d$ mass terms, this symmetry violation does not show up in the ρ–ω sector. The isospin symmetry breaking is shielded by the pion dynamics. Effectively the symmetry is restored by dynamical effects. Here the gluon anomaly plays an important role. The effect of a dynamical symmetry restoration by nonperturbation effects discussed here might be reproduced by lattice simulations. It might be that similar symmetry restoration effects are present in other situations, for example in the electroweak sector, which is sensitive to the dynamics in the TeV region.

I am happy that this paper could be included in the volume dedicated to my friend Hagen Kleinert on the occasion of his 60th birthday. We never collaborated together, but our wordlines met regularly e. g. at CERN, in Berlin, in Pasadena. After introducing Hagen to Dick Feynman, I was happy to learn that they finally wrote a paper together. I have problems seeing Hagen as a sixty–years old colleague, since he appears and acts like a senior post–doc. In Aspen Hagen would never be granted the price–reduction on the

lift–ticket, offered to anyone above 60, unless he takes his birth certificate along – but also in ten years from now Hagen will have the same problem.

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